Benha University	**Electrical Circuits (2)**	**Electrical Eng. Dept.**
Faculty of Engineering		**1st year communication**
Shoubra		**March 2015**

Sheet (5)... Passive Filters

1. Show that a series LR circuit is a low-pass filter if the output is taken across the resistor. Calculate the corner frequency fc if L= 2 mH and R= 10 kΩ.

$$\mathbf{H}(\omega) = \frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{R}{R + j\omega L} = \frac{1}{1 + j\omega L/R}$$

$H(0) = 1$ and $H(\infty) = 0$ showing that this circuit is a lowpass filter.

At the corner frequency, $\left|H(\omega_c)\right| = \frac{1}{\sqrt{2}}$, i.e.

$$\frac{1}{\sqrt{2}} = \frac{1}{\sqrt{1 + \left(\frac{\omega_c L}{R}\right)^2}} \longrightarrow 1 = \frac{\omega_c L}{R} \quad \text{or} \quad \omega_c = \frac{R}{L}$$

Hence,

$$\omega_c = \frac{R}{L} = 2\pi f_c$$

$$f_c = \frac{1}{2\pi} \cdot \frac{R}{L} = \frac{1}{2\pi} \cdot \frac{10 \times 10^3}{2 \times 10^{-3}} = \underline{\mathbf{796\ kHz}}$$

2. Find the transfer function Vo/Vs of the circuit in Figure 1. Show that the circuit is a low-pass filter.

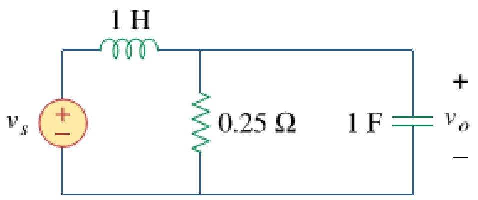

Fig.1

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$$\mathbf{H}(\omega) = \frac{R \parallel \frac{1}{j\omega C}}{j\omega L + R \parallel \frac{1}{j\omega C}}$$

$$\mathbf{H}(\omega) = \frac{\frac{R/j\omega C}{R + 1/j\omega C}}{j\omega L + \frac{R/j\omega C}{R + 1/j\omega C}}$$

$$\mathbf{H}(\omega) = \frac{\mathbf{R}}{\mathbf{R} + \mathbf{j\omega L} - \boldsymbol{\omega}^2\mathbf{RLC}}$$

$H(0) = 1$ and $H(\infty) = 0$ showing that **this circuit is a lowpass filter**.

3. Determine the cutoff frequency of the low-pass filter described by

$$\mathbf{H}(\omega) = \frac{4}{2 + j\omega 10}$$

Find the gain in dB and phase of H (ω) at ω= 2 rad/s.

At dc, $H(0) = \frac{4}{2} = 2$.

Hence, $\left|H(\omega)\right| = \frac{1}{\sqrt{2}} H(0) = \frac{2}{\sqrt{2}}$

$$\frac{2}{\sqrt{2}} = \frac{4}{\sqrt{4 + 100\omega_c^2}}$$

$$4 + 100\omega_c^2 = 8 \longrightarrow \omega_c = 0.2$$

$$H(2) = \frac{4}{2 + j20} = \frac{2}{1 + j10}$$

$$\left|H(2)\right| = \frac{2}{\sqrt{101}} = 0.199$$

In dB, $20\log_{10}\left|H(2)\right| =$ **-14.023**

$\arg H(2) = -\tan^{-1} 10 =$ **-84.3°**

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4. Determine what type of filter in figure 2. Calculate the corner frequency fc

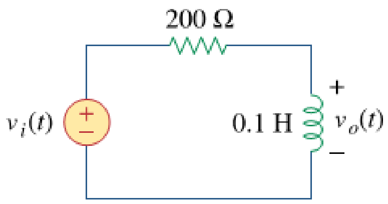

Fig.2

$$\mathbf{H}(\omega) = \frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{j\omega L}{R + j\omega L}$$

$H(0) = 0$ and $H(\infty) = 1$ showing that **<u>this circuit is a highpass filter</u>**.

$$\mathbf{H}(\omega_c) = \frac{1}{\sqrt{2}} = \frac{1}{\sqrt{1 + \left(\frac{R}{\omega_c L}\right)^2}} \longrightarrow 1 = \frac{R}{\omega_c L}$$

or $\omega_c = \frac{R}{L} = 2\pi f_c$

$$f_c = \frac{1}{2\pi} \cdot \frac{R}{L} = \frac{1}{2\pi} \cdot \frac{200}{0.1} = \mathbf{318.3\ Hz}$$

5. In a high-pass RL filter with a cutoff frequency of 100 kHz, L= 40 mH. Find R.

$$\omega_c = \frac{R}{L} = 2\pi f_c$$

$$R = 2\pi f_c L = (2\pi)(10^5)(40 \times 10^{-3}) = \mathbf{25.13\ k\Omega}$$

6. Design a series RLC type band-pass filter with cutoff frequencies of 10 kHz and 11 kHz. Assuming C= 80 pF, find R, L, and Q.

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$$\omega_1 = 2\pi f_1 = 20\pi \times 10^3$$
$$\omega_2 = 2\pi f_2 = 22\pi \times 10^3$$

$$B = \omega_2 - \omega_1 = 2\pi \times 10^3$$
$$\omega_0 = \frac{\omega_2 + \omega_1}{2} = 21\pi \times 10^3$$
$$Q = \frac{\omega_0}{B} = \frac{21\pi}{2\pi} = \underline{\mathbf{10.5}}$$

$$\omega_0 = \frac{1}{\sqrt{LC}} \longrightarrow L = \frac{1}{\omega_0^2 C}$$
$$L = \frac{1}{(21\pi \times 10^3)^2(80 \times 10^{-12})} = \underline{\mathbf{2.872\ H}}$$

$$B = \frac{R}{L} \longrightarrow R = BL$$
$$R = (2\pi \times 10^3)(2.872) = \underline{\mathbf{18.045\ k\Omega}}$$

7. Determine the range of frequencies that will be passed by a series RLC band-pass filter with R= 10Ω, L= 25mH, and C= 0.4 μF. Find the quality factor.

$$\omega_o = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(25 \times 10^{-3})(0.4 \times 10^{-6})}} = 10\ \text{krad/s}$$
$$B = \frac{R}{L} = \frac{10}{25 \times 10^{-3}} = 0.4\ \text{krad/s}$$

$$Q = \frac{10}{0.4} = \underline{\mathbf{25}}$$

$$\omega_1 = \omega_o - B/2 = 10 - 0.2 = 9.8\ \text{krad/s} \quad \text{or} \quad f_1 = \frac{9.8}{2\pi} = 1.56\ \text{kHz}$$
$$\omega_2 = \omega_o + B/2 = 10 + 0.2 = 10.2\ \text{krad/s} \quad \text{or} \quad f_2 = \frac{10.2}{2\pi} = 1.62\ \text{kHz}$$

Therefore,

$$\underline{\mathbf{1.56\ kHz < f < 1.62\ kHz}}$$

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8. The circuit parameters for a series RLC band-stop filter are R= 2 kΩ, L= 0.1 H, C= 40 pF. Calculate:
   (a) The center frequency
   (b) The half-power frequencies
   (c) The quality factor

(a) $$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(0.1)(40\times10^{-12})}} = \underline{\mathbf{0.5\times10^6\ rad/s}}$$

(b) $$B = \frac{R}{L} = \frac{2\times10^3}{0.1} = 2\times10^4$$

$$Q = \frac{\omega_0}{B} = \frac{0.5\times10^6}{2\times10^4} = 25$$

As a high Q circuit,

$$\omega_1 = \omega_0 - \frac{B}{2} = 10^4(50-1) = \underline{\mathbf{490\ krad/s}}$$

$$\omega_2 = \omega_0 + \frac{B}{2} = 10^4(50+1) = \underline{\mathbf{510\ krad/s}}$$

(c) As seen in part (b), $Q = \underline{\mathbf{25}}$

9. Find the bandwidth and center frequency of the band-stop filter shown in figure 3

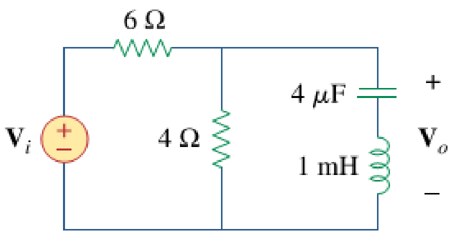
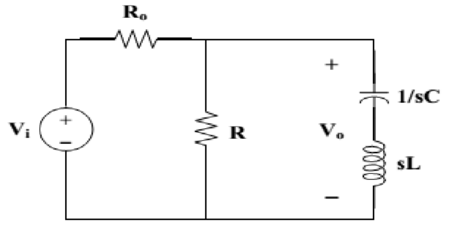


Fig. 3

Consider the circuit below.



$$\mathbf{Z}(s) = R \parallel \left(sL + \frac{1}{sC}\right) = \frac{R\,(sL + 1/sC)}{R + sL + 1/sC}$$

$$\mathbf{Z}(s) = \frac{R\,(1 + s^2LC)}{1 + sRC + s^2LC}$$

$$\mathbf{H} = \frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{\mathbf{Z}}{\mathbf{Z} + R_o} = \frac{R\,(1 + s^2LC)}{R_o + sRR_oC + s^2LCR_o + R + s^2LCR}$$

$$\mathbf{Z}_{in} = R_o + \mathbf{Z} = R_o + \frac{R\,(1 + s^2LC)}{1 + sRC + s^2LC}$$

$$\mathbf{Z}_{in} = \frac{R_o + sRR_oC + s^2LCR_o + R + s^2LCR}{1 + sRC + s^2LC}$$

$s = j\omega$

$$\mathbf{Z}_{in} = \frac{R_o + j\omega RR_oC - \omega^2LCR_o + R - \omega^2LCR}{1 - \omega^2LC + j\omega RC}$$

$$\mathbf{Z}_{in} = \frac{(R_o + R - \omega^2LCR_o - \omega^2LCR + j\omega RR_oC)(1 - \omega^2LC - j\omega RC)}{(1 - \omega^2LC)^2 + (\omega RC)^2}$$

$\text{Im}(\mathbf{Z}_{in}) = 0$ implies that

$$-\omega RC[R_o + R - \omega^2LCR_o - \omega^2LCR] + \omega RR_oC(1 - \omega^2LC) = 0$$

$$R_o + R - \omega^2LCR_o - \omega^2LCR - R_o + \omega^2LCR_o = 0$$

$$\omega^2LCR = R$$

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(1\times10^{-3})(4\times10^{-6})}} = \underline{\mathbf{15.811\ krad/s}}$$

$$\mathbf{H} = \frac{R\,(1 - \omega^2LC)}{R_o + j\omega RR_oC + R - \omega^2LCR_o - \omega^2LCR}$$

$$H_{max} = H(0) = \frac{R}{R_o + R}$$

or $$H_{max} = H(\infty) = \lim_{\omega\to\infty} \frac{R\left(\frac{1}{\omega^2} - LC\right)}{\frac{R_o + R}{\omega^2} + j\frac{RR_oC}{\omega} - LC(R + R_o)} = \frac{R}{R + R_o}$$

At ω_1 and ω_2, $\left|\mathbf{H}\right| = \frac{1}{\sqrt{2}} H_{mzx}$

$$\frac{R}{\sqrt{2}(R_o + R)} = \left| \frac{R(1-\omega^2 LC)}{R_o + R - \omega^2 LC(R_o + R) + j\omega R R_o C} \right|$$

$$\frac{1}{\sqrt{2}} = \frac{(R_o + R)(1-\omega^2 LC)}{\sqrt{(\omega R R_o C)^2 + (R_o + R - \omega^2 LC(R_o + R))^2}}$$

$$\frac{1}{\sqrt{2}} = \frac{10(1-\omega^2 \cdot 4\times10^{-9})}{\sqrt{(96\times10^{-6}\omega)^2 + (10-\omega^2 \cdot 4\times10^{-8})^2}}$$

$$0 = \frac{10(1-\omega^2 \cdot 4\times10^{-9})}{\sqrt{(96\times10^{-6}\omega)^2 + (10-\omega^2 \cdot 4\times10^{-8})^2}} - \frac{1}{\sqrt{2}}$$

$$(10-\omega^2 \cdot 4\times10^{-8})(\sqrt{2}) - \sqrt{(96\times10^{-6}\omega)^2 + (10-\omega^2 \cdot 4\times10^{-8})^2} = 0$$

$$(2)(10-\omega^2 \cdot 4\times10^{-8})^2 = (96\times10^{-6}\omega)^2 + (10-\omega^2 \cdot 4\times10^{-8})^2$$

$$(96\times10^{-6}\omega)^2 - (10-\omega^2 \cdot 4\times10^{-8})^2 = 0$$

$$1.6\times10^{-15}\omega^4 - 8.092\times10^{-7}\omega^2 + 100 = 0$$

$$\omega^4 - 5.058\times10^8 + 6.25\times10^{16} = 0$$

$$\omega^2 = \begin{cases} 2.9109\times10^8 \\ 2.1471\times10^8 \end{cases}$$

Hence,

$$\omega_1 = 14.653 \text{ krad/s}$$

$$\omega_2 = 17.061 \text{ krad/s}$$

$$B = \omega_2 - \omega_1 = 17.061 - 14.653 = \underline{\mathbf{2.408\ krad/s}}$$